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Prediction of Ice/Frost Growth on Insulated Cryogenic Tanks

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Introduction

WHEN the Space Shuttle external tank (ET) is filled with liquid cryogenics (hydrogen and oxygen), the temperature at certain locations on the outer surface of the spray-on foam insulation (SOFI) can, under certain weather conditions, fall below freezing. When this happens there is a possibility of frost or ice accumulation. If ice or sufficiently dense frost breaks loose during ascent it may strike and damage thermal protective tiles on the orbiter.

The work reported here involved development of a computer model for predicting frost and ice formation on the ET under a variety of weather conditions. The model, however, is applicable to any insulated cryogenic tank for which the heat- and mass-transfer coefficients can be determined.

Heat Balance

The rate of frost or ice formation is governed by the rate of heat transport to the frost/ice surface through the moist air boundary layer, and by the rate at which heat is transported through the frost/ice layer and thermal insulation to the cryogenically cooled wall. In this analysis it is assumed that the transport is one dimensional (normal to wall) and that the time scale for transport is small compared to the time scale for changes in temperature profile. Thus a quasi-steady-state situation prevails for which temperature is independent of time over small time intervals, during which changes in frost/ice thickness and density are computed. The heat flux q is constant through the moist air boundary layer, frost/ice layer, and SOFI during this time interval.

Heat is transported to the tank surface from the environment by convection and radiational exchange. In addition, heat content is carried to the surface from the environment by water vapor diffusion. The heat flux to the surface can be expressed as the sum of these effects by

$$q = q_{\mathrm{conv}} + q_{\mathrm{lat}} + q_{\mathrm{rad}} \tag{1}$$

Received July 12, 1982; revision received Nov. 19, 1982.

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The first term on the right side of Eq. (1) represents the convective heat flux. The second term represents the latent heat released at the surface due to condensation followed by freezing (or direct frost deposition) of water vapor. The last term on the right side of Eq. (1) represents the radiation exchange between the environment and the frost surface. This is expressed as the sum of the long wave radiation exchange with the sky, the long wave radiation exchange with the Earth's surface, and the absorbed solar radiation.

Within the frost layer, thermal transport occurs by conduction and the latent heat associated with water vapor diffusion. The transport is represented by using an effective frost thermal conductivity, k_e in the Fourier heat conduction law, where k_e is given by the Brailford random mixture model.[1] This results in

$$q = (1/f)\int_{T_s}^{T_f} k_e \mathrm{d}T \tag{2}$$

where f is the thickness of the frost layer, T_f is the temperature at the air/frost interface, and T_s is the temperature at the frost/insulation interface. The thermal conductivity in Eq. (2) approaches the value for bulk ice as the frost density approaches ice density, and it approaches the thermal conductivity of air as the frost density approaches air density.

In the thermal insulation, heat transport occurs solely by conduction. The temperature dependence of the thermal conductivity for the insulation is assumed to be linear. When substituted into the Fourier heat conduction law, integration leads to

$$q = \frac{1}{s}\left\{\frac{a}{2}(T_s^2 - T_w^2) + b(T_s - T_w)\right\} \tag{3}$$

where s is the thickness of the insulation and T_w is the temperature of the cryogenic wall beneath the insulation. Values for the constants a and b are determined from the temperature dependence of the thermal conductivity of the insulation.

The set of Eqs. (1-3) can be solved for the frost surface temperature T_f and insulation surface temperature T_s if the heat- and mass-transfer coefficients (C_T and C_ρ) are known. Appropriate values for C_T and C_ρ are discussed in the next section. Because the latent heat release at the surface and the surface temperature are interactive and Eqs. (1-3) are nonlinear, it is necessary to use an iterative approach to finding the correct surface temperature and associated latent heat release.

Determination of Heat- and Mass-Transfer Coefficients

Local heat- and mass-transfer coefficients are required as input to the model for computing frost and icing rates. Values for these coefficients are required for both laminar and turbulent flow resulting from either free or forced convection and should take into account the surface roughness.

There is presently little experimental data on heat and mass transfer for turbulent free convective flow over rough surfaces. In order to assess the significance of the insulation surface roughness, it was decided to develop a free convection boundary-layer model that included the effects of roughness. The physics of this boundary-layer model are discussed in detail in Ref. 2. It is based on the method of Kato et al.[3] The work of Kato et al. applies to heat transfer by free convection over a smooth surface. The technique was extended to apply to situations involving both heat and mass transport. The effect of roughness is taken into account through the eddy diffusivities, as proposed by Street[4] and Cebeci and Chang.[5] Two parameters are required for characterizing the surface roughness: the geometric roughness height κ and the equivalent sand grain roughness κ_s. κ and κ_s are related through the correlation proposed by Dirling.[6]

For forced convection over a right circular cylinder with axis normal to the flow, the local Nusselt number is expressed by

$$Nu = (Re_d \cos\theta)^{1/2} \tag{4}$$

if the flow is laminar. Re_d is the Reynolds number based on the cylinder diameter and θ is the angular distance from the stagnation line. Equation (4) provides a good fit with measurements performed by Schmidt and Wenner[7] and with the theory of Froessling and Dienemann[7] for $\theta \leq 80$ deg.

For turbulent flow the following empirical expression is used[8]:

$$Nu = \sqrt{Re_d}\,[2.4 + 1.2\sin 3.6(\theta - 0.44] \tag{5}$$

The criteria for determining transition from laminar to turbulent flow are based on flat plate experiments. This leads to transition at a local Reynolds number of about 3.4×10^5. The local heat-transfer coefficient for forced convection is then determined from the definition of the Nusselt number and the local mass-transfer coefficient is found using the Chilton-Colburn analogy.

Ice/Frost Density and Thickness

Changes in ice/frost thickness and frost density are calculated during small time intervals that are selected by the user. The time interval must be sufficiently small so that the change in frost surface temperature does not significantly change the heat balance. (This is necessary to ensure validity of the quasi-steady-state assumption of the model.) Validity is normally assured if the time interval is less than 1 min during the first 5 min of frost growth, and 5 min thereafter.

If the surface temperature is determined to be below the dew point temperature and equal to the freezing temperature, water freezes on the surface in the form of ice. Under these conditions the ice thickness is updated from the calculated water vapor mass flux to the surface using a nominal value of 917 kg/m^3 for the density of ice.

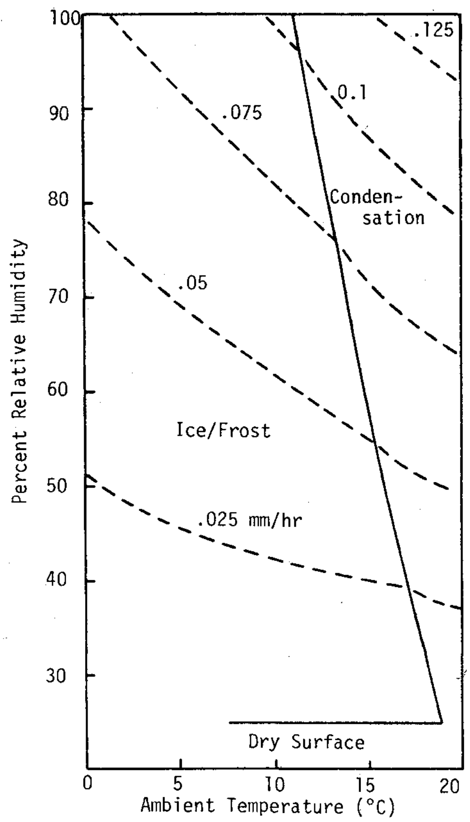

Fig. 1 Regimes for ice/frost and condensation on external tank and lines of constant ice growth rate (clear night sky with no wind).

If the surface temperature is determined to be below both the dew (frost) point temperature and the freezing temperature, water vapor is assumed to be deposited on the surface in the form of frost. Because of the porous nature of frost, it is possible for water vapor diffusion to occur within the developing frost layer. The result of this diffusion is that the density of the frost layer changes with time, and this time dependency must be determined. Details of the numerical procedure employed are described in Ref. 2 and will not be repeated here.

Results of Predicted External Tank Surface Temperatures and Ice/Frost Formation Rates

A number of computations were made utilizing the computer model in order to gain information on the meteorological conditions under which frost or ice can form on the external tank (ET) of the Space Shuttle and the severity of icing that might be encountered.

Figure 1 summarizes the influence of ambient temperature and relative humidity on predicted ET icing rates as a result of water vapor flux to the surface for clear night skies with no wind. For ambient conditions representative of the upper right-hand region of the figure, the surface temperature is consistently above freezing and condensation occurs. Below about 25% relative humidity the surface temperature is above the dew point and the surface is dry. In the upper left-hand region of the figure some portion of the tank surface is at or below freezing and frost or ice accumulates. The maximum rate of ice or water film accumulation is indicated by the dashed lines in the figure.

Summary and Conclusions

The investigations carried out during the course of this program show that it is feasible to predict ET surface temperatures to an accuracy of about 1°C, and to predict frost thicknesses with an error of less than 25%. The error in predicted ice thicknesses is less than that for frost because of the greater certainty of the values for density and thermal conductivity. Analysis indicates that water vapor transport from the air to the ET surface results in maximum icing rates that are smaller than those required for significant accumulation over a 4-h period.

As a result of variations in SOFI thickness, however, it is possible for water to condense on warmer (thicker) portions of the SOFI surface and run down to colder (thinner) portions where it freezes. This run-down mechanism could result in local icing rates that are considerably greater than those resulting from vapor transport alone.

Acknowledgments

This work was supported by the U.S. Air Force, Space Division, under Contract F04701-81-K-0002. Lt. Mark Nussmeier served as technical monitor for the Air Force.

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